Crystallization of Compound Plastic Optical Fibers

J.I. Ramos and Francisco J. Blanco–Rodríguez

Escuela de Ingenierías
Universidad de Málaga

Fifth International Conference on Thermal Engineering: Theory and Applications
May 10–14, 2010, Marrakesh, Morocco

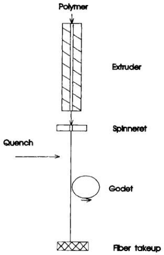
Index

Introduction

- Polymer Optical Fibers (POF) are manufactured by MELT SPINNING processes.
- Necessary: modelling of the drawing process for both hollow and solid compound optical fibers.
- Previous studies are based on one–dimensional models.
- NO INFORMATION ABOUT RADIAL VARIATIONS.
- Use of a hybrid model for melt spinning phenomena.
- Applications
  1. Telecommunications: Data transmission.
  2. Chemical industry: Filtration and separation processes.
  3. Biomedical industry.
  4. Textile industry.

Melt spinning

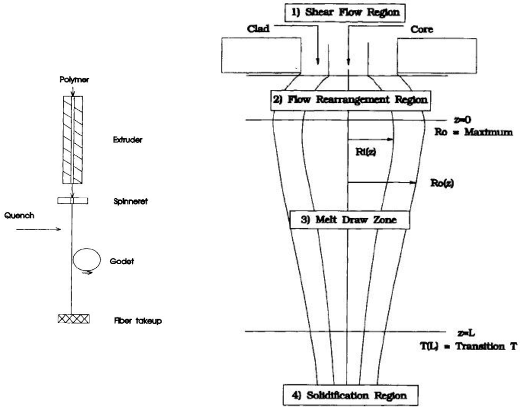
Involves the extrusion and drawing of a polymer cylinder. Four zones.



1. Shear Flow Region.
2. Flow Rearrangement Region.
3. **Melt Drawing Zone.**
4. **Solidification region.**

Melt spinning

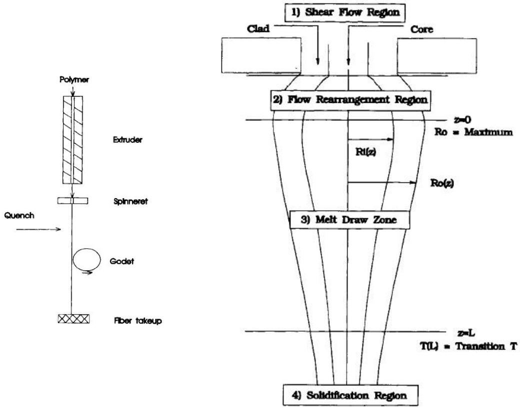
Involves the extrusion and drawing of a polymer cylinder. Four zones.



1. Shear Flow Region.
2. Flow Rearrangement Region.
3. **Melt Drawing Zone.**
4. **Solidification region.**

Melt spinning

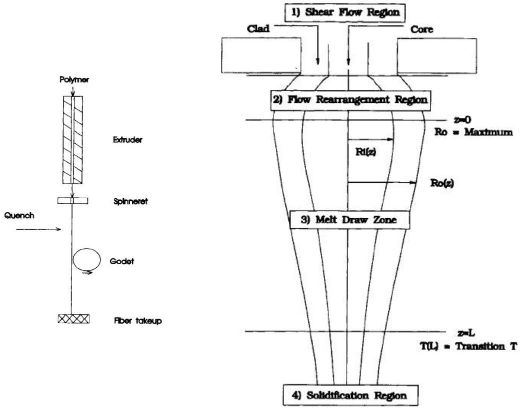
Involves the extrusion and drawing of a polymer cylinder. Four zones.



1. Shear Flow Region.
2. Flow Rearrangement Region.
3. Melt Drawing Zone.
4. Solidification region.

Melt spinning

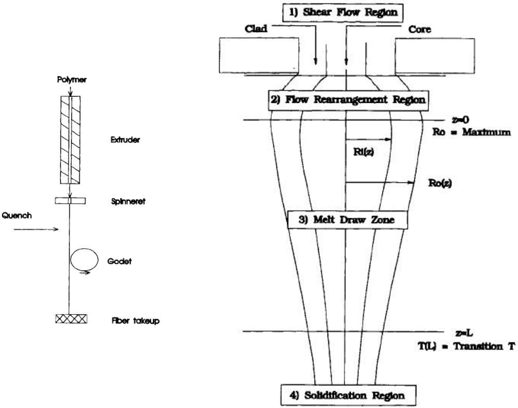
Involves the extrusion and drawing of a polymer cylinder. Four zones.



1. Shear Flow Region.
2. Flow Rearrangement Region.
3. Melt Drawing Zone.
4. Solidification region.

Melt spinning

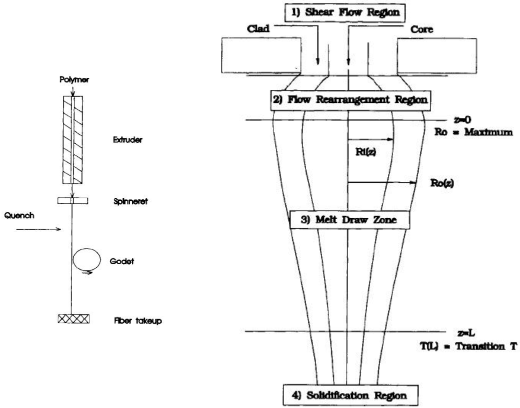
Involves the extrusion and drawing of a polymer cylinder. Four zones.



1. Shear Flow Region.
2. Flow Rearrangement Region.
3. **Melt Drawing Zone.**
4. Solidification region.

Melt spinning

Involves the extrusion and drawing of a polymer cylinder. Four zones.



1. Shear Flow Region.
2. Flow Rearrangement Region.
3. **Melt Drawing Zone.**
4. **Solidification region.**

Problem formulation (I)

- **Mass conservation equation**

$$\nabla \cdot \mathbf{v}_i = 0 \qquad i = 1, 2,$$

where $\mathbf{v} = u(r,x)\,\mathbf{e_x} + v(r,x)\,\mathbf{e_r}$

- **Linear Momentum conservation equation**

$$\rho_i \left(\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i \right) = -\nabla p + \nabla \cdot \boldsymbol{\tau}_i + \rho_i \cdot \mathbf{f}^m \qquad i = 1, 2,$$

where $\mathbf{f}^m = g\,\mathbf{e_x}$

- **Energy conservation equation**

$$\rho_i\,C_i \left(\frac{\partial T_i}{\partial t} + \mathbf{v}_i \cdot \nabla T_i \right) = -\nabla \cdot \mathbf{q}_i, \qquad i = 1, 2,$$

- **Constitutive equations**
  - Newtonian rheology

$$\boldsymbol{\tau}_i = 2\mu_{eff,i}\boldsymbol{D}_i = \mu_{eff,i}\left(\nabla \mathbf{v}_i + \nabla \mathbf{v}_i^T\right),$$

  - Fourier's law

$$\mathbf{q}_i = -k_i\,\nabla T_i,$$

Problem formulation (II)

- **Molecular orientation: Doi–Edwards equation**

$$\frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S = -\frac{\phi}{\lambda} F(S) + G(\nabla \mathbf{v}, S),$$
$$F(S) = S\ (1 - N/3\ (1 - S)\,(2\,S + 1))$$
$$G(\nabla \mathbf{v}, S) = (1 - S)\,(2\,S + 1)\,\frac{\partial u}{\partial x}.$$

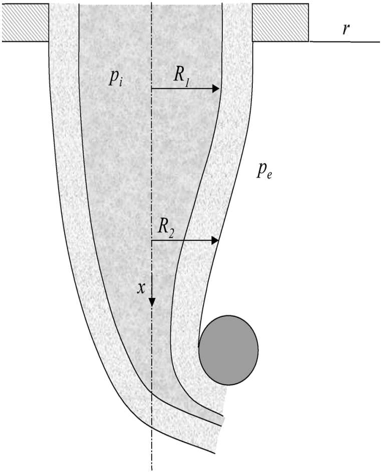
- **Crystallization: Avrami–Kolmogorov kinetics**

$$\frac{\partial \theta_i}{\partial t} + \mathbf{v} \cdot \nabla \theta_i = k_{Ai}(\mathcal{S}_i)\,(\theta_{\infty\, i} - \theta_i)\,, \qquad i = 1, 2,$$

where

$$k_{Ai}(S_i) = k_{Ai}(0) \exp\left(a_{2i} S_i^2\right), \qquad i = 1, 2.$$

Problem formulation (III)



Kinematic, dynamic and thermal boundary conditions are required:

- Symmetry conditions $(r = 0)$
- Die exit conditions $(x = 0)$
- Take–up point conditions $(x = L)$
- Conditions on free surfaces of compound fiber $(r = R_1(x)$ and $r = R_2(x))$

Non–dimensionalize

- **Non–dimensional variables**

$$\hat{t} = \frac{t}{L/u_0} \qquad \hat{r} = \frac{r}{R_0} \qquad \hat{x} = \frac{x}{L} \quad \Rightarrow \quad \epsilon = \frac{R_0}{L}$$

$$\hat{u} = \frac{u}{u_0} \qquad \hat{v} = \frac{v}{(u_0\,\epsilon)} \qquad \hat{p} = \frac{p}{(\mu_0 u_0/L)} \qquad \hat{T} = \frac{T}{T_0}$$

$$\hat{\rho} = \frac{\rho}{\rho_0} \qquad \hat{C} = \frac{C}{C_0} \qquad \hat{\mu} = \frac{\mu}{\mu_0} \qquad \hat{k} = \frac{k}{k_0}$$

- **Non–dimensional numbers**

$$Re = \frac{\rho_0 u_0 R_0}{\mu_0}, \quad Fr = \frac{u_0^2}{g R_0}, \quad Ca = \frac{\mu_0 u_0}{\sigma_2},$$

$$Pe = \frac{\rho_0 C_0}{k_0}\, u_0 R_0, \quad Bi = \frac{h R_0}{k_0}$$

- **Perturbation method using the fiber slenderness** $(\epsilon << 1)$

$$\Psi_i = \Psi_{i,0} + \epsilon^2 \Psi_{i,2} + O(\epsilon^4)\,,$$

for the variables $\hat{R}_i$, $\hat{u}_i$, $\hat{v}_i$, $\hat{p}_i$ and $\hat{T}_i$ where $i = 1, 2$.

- **Steady–state flow regime considered**

$$Re = \epsilon\,\bar{R}, \qquad Fr = \frac{\bar{F}}{\epsilon}, \qquad Ca = \frac{\bar{C}}{\epsilon},$$

$$Pe = \epsilon\,\bar{P}, \qquad Bi = \epsilon^2\,\bar{B}$$

where $\bar{\Upsilon} = O(1)$.

One–dimensional equations of the $1 + 1/2$D model

- **Asymptotic one–dimensional mass conservation equation**

$$\frac{d}{d\hat{x}}\left(\mathcal{A}_i\,\hat{U}\right) = 0, \qquad i = 1, 2,$$

$$\mathcal{A}_1 = \frac{\hat{R}_1^2}{2}, \qquad \mathcal{A}_2 = \frac{\hat{R}_2^2 - \hat{R}_1^2}{2},$$

- **Asymptotic one–dimensional linear momentum equation**

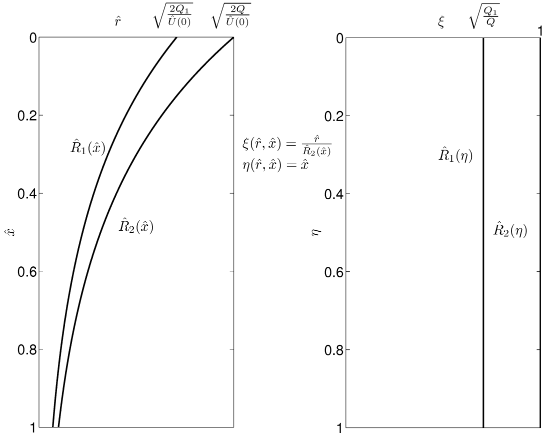
$$\begin{aligned}\bar{R}(\hat{\rho}_1\mathcal{A}_1 + \hat{\rho}_2\mathcal{A}_2)\hat{U}\,\frac{d\hat{U}}{d\hat{x}} &= \frac{d}{d\hat{x}}\left(3\left(<\hat{\mu}_{eff,1}>\mathcal{A}_1 + <\hat{\mu}_{eff,2}>\mathcal{A}_2\right)\frac{d\hat{U}}{d\hat{x}}\right)\\ &+ \frac{1}{2\bar{C}}\left(\frac{d\hat{R}_2}{d\hat{x}} + \frac{\sigma_1}{\sigma_2}\frac{d\hat{R}_1}{d\hat{x}}\right)\\ &+ (\hat{\rho}_1\mathcal{A}_1 + \hat{\rho}_2\mathcal{A}_2)\frac{\bar{R}}{\bar{F}}\end{aligned}$$

- **Effective dynamic viscosity**

$$\hat{\mu}_{eff,i} = \hat{G}_i\,\exp\left(\hat{E}_i\left(1 - \hat{T}_i\right) + \beta_i\left(\frac{\theta_i}{\theta_{\infty,i}}\right)^{n_i}\right) + \frac{2}{3}\,\alpha_i\,\lambda_i\,S_i^2, \qquad i = 1, 2.$$

Mapping: 2D model

$(\hat{r}, \hat{x}) \mapsto (\xi, \eta)$ maps $\Omega_{\hat{r}\hat{x}} = \left\{[0,\ \hat{R}_2(\hat{x})] \times [0,\ 1]\right\}$ into a rectangular domain $\Omega_{\xi\eta} = \{[0,\ 1] \times [0,\ 1]\}$



Two–dimensional equations of the $1 + 1/2D$ model

- **Two–dimensional energy equation**

$$\frac{\partial \hat{T}_i}{\partial \eta} = \frac{1}{2\,Q}\frac{1}{\bar{P}_i}\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \hat{T}_i}{\partial \xi}\right) \qquad i = 1, 2,$$

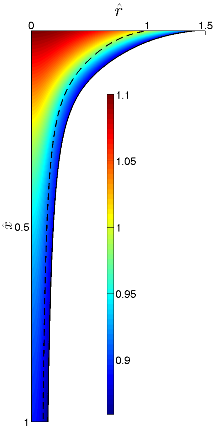
- **Two–dimensional molecular orientation parameter equation**

$$\hat{U}\,\frac{\partial S_i}{\partial \eta} \;=\; -\frac{\phi_i}{\lambda_i}\,S_i\,\left(1 - N_i/3\,(1 - S_i)\,(2\,S_i + 1)\right) \;+\; (1 - S_i)\,(2\,S_i + 1)\,\frac{d\hat{U}}{d\eta}, \qquad i = 1, 2.$$

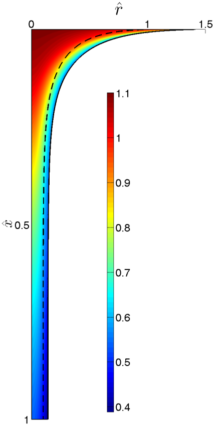
- **Two-dimensional degree of crystallinity equation**

$$\hat{U}\,\frac{\partial \theta_i}{\partial \eta} = k_{Ai}(0)\exp\left(a_{2i}S_i^2\right)\,\left(\theta_{\infty,i} - \theta_i\right), \qquad i = 1, 2,$$

Influence of Biot number on cooling process

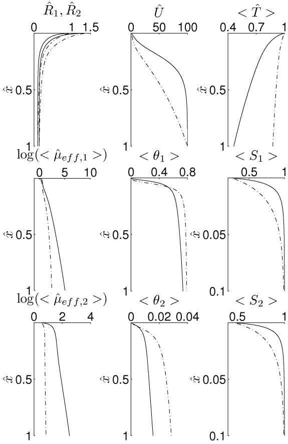


$\bar{B} = 0{,}5$



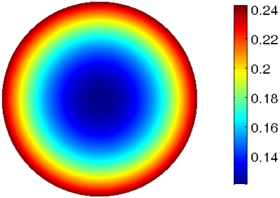
$\bar{B} = 5{,}0$

Influence of Biot number on the radially averaged results

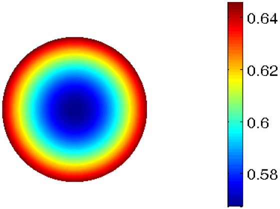


$(- \cdot -)$ $\bar{B} = 0{,}5$ and $(-)$ $\bar{B} = 5{,}0$

Degree of crystallinity near the die exit



$\hat{x} = 0{,}04$



$\hat{x} = 0{,}10$

Discussion

Contributions of the present work:

1. Development of a $1 + 1/2$D model for both amorphous and semicrystalline fibers with Newtonian rheology.
2. Validation of applicability range of the 1D model with the $1 + 1/2$D one.
3. Determination of the two–dimensional fields of temperature, molecular orientation parameter and degree of crystallinity for solid compound fibers.
4. Find substantial temperature non–uniformities (affect the degree of crystallization and have great effects on the properties of compound fibers) in the radial direction exist even at small Biot numbers.

About the authors...

Francisco J. Blanco–Rodríguez
e-mail: fjblanco@lcc.uma.es
website: `http://www.lcc.uma.es/~fjblanco`

J. I. Ramos
e-mail: jirs@lcc.uma.es

Document created by LaTeX(Beamer class).

THANK YOU FOR YOUR ATTENTION