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TWO-DIMENSIONAL ANALYSIS OF THE CRYSTALLIZATION OF HOLLOW COMPOUND PLASTIC FIBERS

Francisco J. Blanco-Rodríguez, J. I. Ramos

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29071 Málaga, Spain*

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 - ② Textile industry.
 - ③ Optics: Data transmission.
 - ④ Biomedical industry.
- Necessary: modelling of the drawing process of semi-crystalline hollow compound fibers.
- Previous studies are based on one-dimensional models of amorphous, slender fibers at very low Re and Bi numbers.
- NO INFORMATION ABOUT TEMPERATURE NON-UNIFORMITIES IN THE RADIAL DIRECTION.
- Use of a two-dimensional model for fiber spinning.



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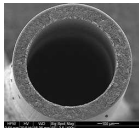
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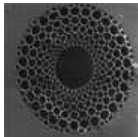
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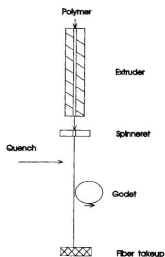
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Involves the extrusion and drawing of a polymer cylinder. Four zones.



- ① Shear Flow Region.
- ② Flow Rearrangement Region.
- ③ Melt Drawing Zone.
- ④ Solidification region.

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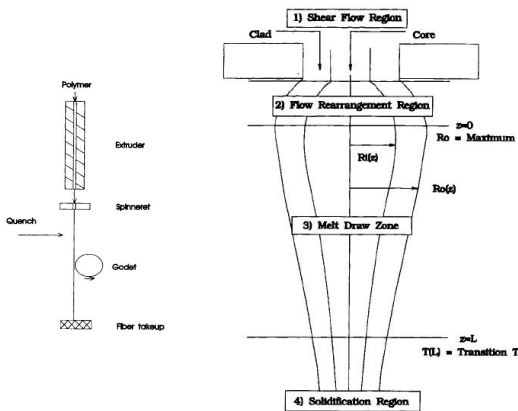
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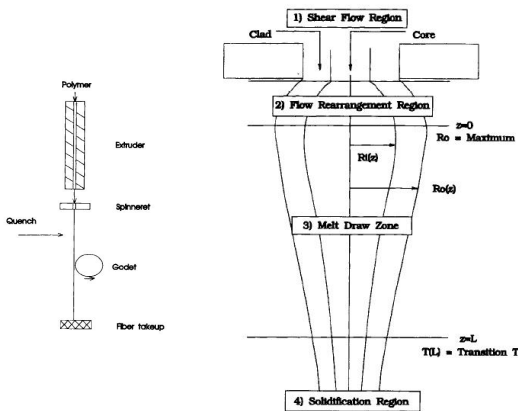
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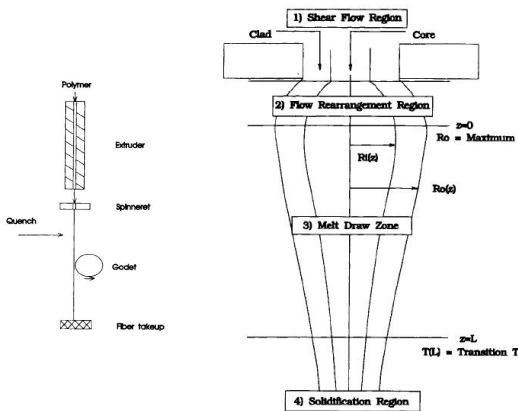
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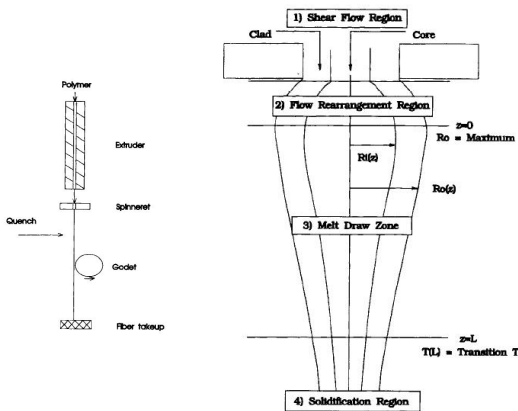
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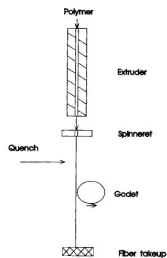
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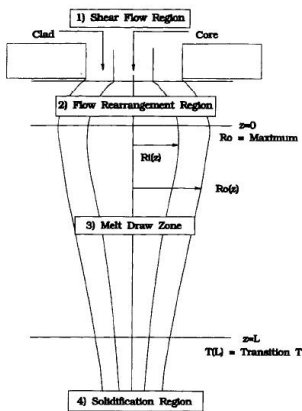
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- Mass conservation equation

$$\nabla \cdot \mathbf{v}_i = 0 \quad i = 1, 2,$$

where $\mathbf{v} = u(r, x) \mathbf{e}_x + v(r, x) \mathbf{e}_r$

- Linear Momentum conservation equation

$$\rho_i \left(\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i \right) = -\nabla p_i + \nabla \cdot \boldsymbol{\tau}_i + \rho_i \cdot \mathbf{f}^m \quad i = 1, 2,$$

where $\mathbf{f}^m = g \mathbf{e}_x$

- Energy conservation equation

$$\rho_i C_i \left(\frac{\partial T_i}{\partial t} + \mathbf{v}_i \cdot \nabla T_i \right) = -K_i \Delta T_i \quad i = 1, 2,$$

- Constitutive equations

- Rheology

$$\boldsymbol{\tau} = \mu (\nabla \mathbf{v} + \nabla \mathbf{v}^T) + \boldsymbol{\tau}_p,$$

where

$$\boldsymbol{\tau}_p = 3c k_B T \left[-\frac{\lambda}{\phi} F(\mathbf{S}) + 2\lambda (\nabla \mathbf{v}^T : \mathbf{S}) (\mathbf{S} + \mathbf{I}/3) \right]$$



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- Crystallization: Avrami–Kolmogorov’s theory & Ziabicki’s model

$$\frac{\partial Y_i}{\partial t} + \mathbf{v}_i \cdot \nabla Y_i = k_{Ai}(\mathcal{S}_i) (Y_{\infty,i} - Y_i) \quad i = 1, 2,$$

where

$$k_{Ai}(\mathcal{S}_i) = k_{Ai}(0) \exp(a_{2i} \mathcal{S}_i^2), \quad i = 1, 2.$$

- Molecular orientation scalar order parameter

$$S \equiv \sqrt{\frac{3}{2} (\mathbf{S} : \mathbf{S})} \quad \mathbf{S} = \frac{1}{3} \cdot \text{diag} (S_{rr}, -(S_{rr} + S_{xx}), S_{xx}),$$

- Molecular orientation tensor equation: Doi–Edwards theory

$$\mathbf{S}_{(1)} = F(\mathbf{S}) + G(\nabla \mathbf{v}, \mathbf{S}),$$

$$F(\mathbf{S}) = -\frac{\phi}{\lambda} \{ (1 - N/3) \mathbf{S} - N (\mathbf{S} \cdot \mathbf{S}) + N (\mathbf{S} : \mathbf{S}) (\mathbf{S} + \mathbf{I}/3) \}$$

$$G(\nabla \mathbf{v}, \mathbf{S}) = \frac{1}{3} (\nabla \mathbf{v} + \nabla \mathbf{v}^T) - 2 (\nabla \mathbf{v}^T : \mathbf{S}) (\mathbf{S} + \mathbf{I}/3).$$

where subscript (1) denote UCTD operator

$$\Lambda_{(1)} = \frac{\partial \Lambda}{\partial t} + \mathbf{v} \cdot \nabla \Lambda - (\nabla \mathbf{v}^T \cdot \Lambda + \Lambda \cdot \nabla \mathbf{v})$$



Molecular orientation and crystallinity models

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About the
authors...

- Crystallization: Avrami–Kolmogorov’s theory & Ziabicki’s model

$$\frac{\partial Y_i}{\partial t} + \mathbf{v}_i \cdot \nabla Y_i = k_{Ai}(\mathcal{S}_i) (Y_{\infty,i} - Y_i) \quad i = 1, 2,$$

where

$$k_{Ai}(\mathcal{S}_i) = k_{Ai}(0) \exp(a_{2i}\mathcal{S}_i^2), \quad i = 1, 2.$$

- Molecular orientation scalar order parameter

$$S \equiv \sqrt{\frac{3}{2}(\mathbf{S} : \mathbf{S})} \quad \mathbf{S} = \frac{1}{3} \cdot \text{diag}(S_{rr}, -(S_{rr} + S_{xx}), S_{xx}),$$

- Molecular orientation tensor equation: Doi–Edwards theory

$$\mathbf{S}_{(1)} = F(\mathbf{S}) + G(\nabla \mathbf{v}, \mathbf{S}),$$

$$F(\mathbf{S}) = -\frac{\phi}{\lambda} \{ (1 - N/3) \mathbf{S} - N(\mathbf{S} \cdot \mathbf{S}) + N(\mathbf{S} : \mathbf{S})(\mathbf{S} + \mathbf{I}/3) \}$$

$$G(\nabla \mathbf{v}, \mathbf{S}) = \frac{1}{3} (\nabla \mathbf{v} + \nabla \mathbf{v}^T) - 2 (\nabla \mathbf{v}^T : \mathbf{S}) (\mathbf{S} + \mathbf{I}/3).$$

where subscript (1) denote UCTD operator

$$\Lambda_{(1)} = \frac{\partial \Lambda}{\partial t} + \mathbf{v} \cdot \nabla \Lambda - (\nabla \mathbf{v}^T \cdot \Lambda + \Lambda \cdot \nabla \mathbf{v})$$

Boundary and initial conditions

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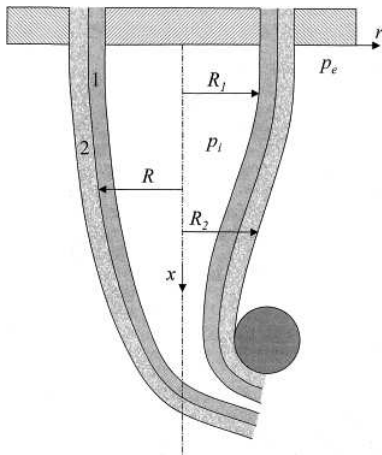
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Kinematic, dynamic and thermal boundary conditions are required:

- Initial conditions ($t = 0$)
- Die exit conditions ($x = 0$)
- Take-up point conditions ($x = L$)
- Conditions on free surfaces of hollow compound fiber ($r = R_1(x)$, $r = R(x)$ and $r = R_2(x)$)

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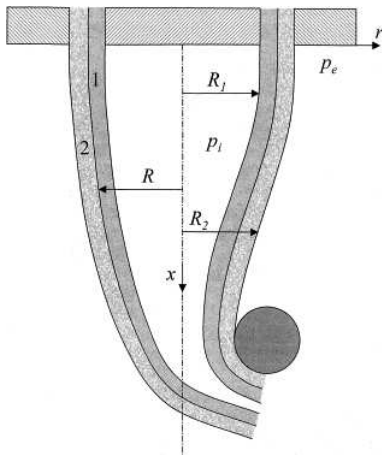
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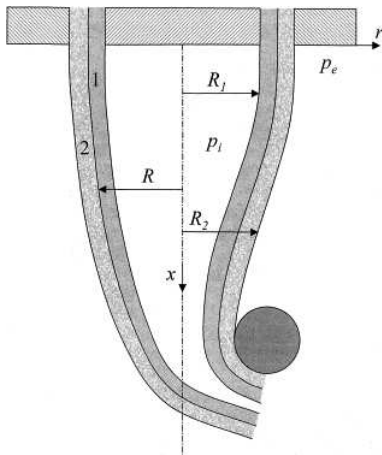
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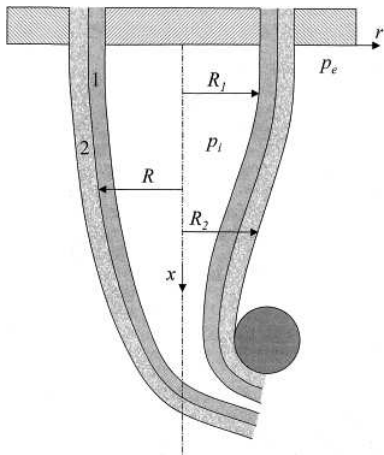
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- Non-dimensional variables

$$\hat{t} = \frac{t}{(L/u_0)} \quad \hat{r} = \frac{r}{R_0} \quad \hat{x} = \frac{x}{L} \quad \Rightarrow \quad \epsilon = \frac{R_0}{L}$$

$$\hat{u} = \frac{u}{u_0} \quad \hat{v} = \frac{v}{(u_0 \epsilon)} \quad \hat{p} = \frac{p}{(\mu_0 u_0 / L)} \quad \hat{T} = \frac{T}{T_0}$$

$$\hat{\rho} = \frac{\rho}{\rho_0} \quad \hat{C} = \frac{C}{C_0} \quad \hat{\mu} = \frac{\mu}{\mu_0} \quad \hat{K} = \frac{K}{K_0}$$

- Non-dimensional numbers

$$Re = \frac{\rho_0 u_0 R_0}{\mu_0}, \quad Fr = \frac{u_0^2}{g R_0}, \quad Ca = \frac{\mu_0 u_0}{\sigma},$$

$$Pr = \frac{\mu_0 C_0}{K_0}, \quad Pe = Re Pr, \quad Bi = \frac{h R_0}{K_0}$$



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- **Asymptotic method using the fiber slenderness, $\epsilon \ll 1$, as perturbation parameter**

$$\Psi_i = \Psi_{i,0} + \epsilon^2 \Psi_{i,2} + O(\epsilon^4),$$

for the variables \hat{R}_i , \hat{u}_i , \hat{v}_i , \hat{p}_i and \hat{T}_i where $i = 1, 2$.

- **Flow regime steady ($\frac{\partial}{\partial t} = 0$) jets and**

$$Re = \epsilon \bar{R}, \quad Fr = \frac{\bar{F}}{\epsilon}, \quad Ca = \frac{\bar{C}}{\epsilon},$$

$$Pe = \epsilon \bar{P}, \quad Bi = \epsilon^2 \bar{B}$$

where $\bar{\Upsilon} = O(1)$.



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- **Flow regime steady ($\frac{\partial}{\partial \tilde{t}} = 0$) jets and**

$$Re = \epsilon \bar{R}, \quad Fr = \frac{\bar{F}}{\epsilon}, \quad Ca = \frac{\bar{C}}{\epsilon},$$

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- Asymptotic one-dimensional mass conservation equation

$$\mathcal{A}_1 \mathcal{B} = V_1,$$

$$\mathcal{A}_2 \mathcal{B} = V_2,$$

$$\frac{d}{d\hat{x}} \left(\frac{\mathcal{R}_0^2}{2} \mathcal{B} \right) = \mathcal{C}(\hat{x})$$

where

$$\mathcal{A}_1 = \frac{\mathcal{R}_0^2 - \mathcal{R}_1^2}{2}, \quad \mathcal{A}_2 = \frac{\mathcal{R}_2^2 - \mathcal{R}_0^2}{2},$$

and

$$\mathcal{C}(\hat{x}) = \left(\frac{1}{2\bar{C}} \right) \frac{\left(\frac{\sigma_1}{\sigma} \right) \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_0} + \left(\frac{\sigma_2}{\sigma} \right) \frac{1}{\mathcal{R}_2}}{\langle \hat{\mu}_{e1,0} \rangle \left(\frac{1}{\mathcal{R}_0^2} - \frac{1}{\mathcal{R}_1^2} \right) + \langle \hat{\mu}_{e2,0} \rangle \left(\frac{1}{\mathcal{R}_2^2} - \frac{1}{\mathcal{R}_0^2} \right)},$$



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- Asymptotic one-dimensional axial momentum equation

$$\begin{aligned}(\hat{\rho}_1 V_1 + \hat{\rho}_2 V_2) \bar{R} \frac{dB}{d\hat{x}} &= (\hat{\rho}_1 \mathcal{A}_1 + \hat{\rho}_2 \mathcal{A}_2) \frac{\bar{R}}{\bar{F}} \\ &+ \frac{d}{d\hat{x}} \left(3 \langle \hat{\mu}_{e1,0} \rangle \mathcal{A}_1 + \langle \hat{\mu}_{e2,0} \rangle \mathcal{A}_2 \right) \frac{dB}{d\hat{x}} \\ + 2\mathcal{C}(\hat{x}) \left(-\frac{\langle \hat{\mu}_{e1,0} \rangle}{\mathcal{R}_1} \frac{d\mathcal{R}_1}{d\hat{x}} + \frac{\langle \hat{\mu}_{e1,0} \rangle - \langle \hat{\mu}_{e2,0} \rangle}{\mathcal{R}_0} \frac{d\mathcal{R}_0}{d\hat{x}} + \frac{\langle \hat{\mu}_{e2,0} \rangle}{\mathcal{R}_2} \frac{d\mathcal{R}_2}{d\hat{x}} \right) \\ &- \left(\mathcal{A}_1 \frac{d\mathcal{D}_1}{d\hat{x}} + \mathcal{A}_2 \frac{d\mathcal{D}_2}{d\hat{x}} \right),\end{aligned}$$

where

$$\mathcal{D}_1 = -\langle \hat{\mu}_{e1,0} \rangle \frac{2\mathcal{C}(\hat{x})}{\mathcal{R}_1^2} - \frac{1}{\bar{C}} \left(\frac{\sigma_1}{\sigma} \right) \frac{1}{\mathcal{R}_1},$$

$$\mathcal{D}_2 = -\langle \hat{\mu}_{e2,0} \rangle \frac{2\mathcal{C}(\hat{x})}{\mathcal{R}_2^2} + \frac{1}{\bar{C}} \left(\frac{\sigma_2}{\sigma} \right) \frac{1}{\mathcal{R}_2}$$

- Incompressibility condition

$$\mathcal{V}(\hat{r}, \hat{x}) = \frac{\mathcal{C}(\hat{x})}{\hat{r}} - \frac{\hat{r}}{2} \frac{dB}{d\hat{x}},$$



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- Incompressibility condition

$$\mathcal{V}(\hat{r}, \hat{x}) = \frac{\mathcal{C}(\hat{x})}{\hat{r}} - \frac{\hat{r}}{2} \frac{d\mathcal{B}}{d\hat{x}},$$



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• Two-dimensional energy equation

$$\mathcal{B} \frac{\partial \hat{T}_1}{\partial \hat{x}} + \left(\frac{\mathcal{C}(\hat{x})}{\hat{r}} - \frac{\hat{r}}{2} \frac{d\mathcal{B}}{d\hat{x}} \right) \frac{\partial \hat{T}_1}{\partial \hat{r}} = \frac{1}{P_1} \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left(\hat{r} \frac{\partial \hat{T}_1}{\partial \hat{r}} \right) \quad \mathcal{R}_1 \leq \hat{r} \leq \mathcal{R}_0,$$

$$\hat{T}_1(\hat{r}, 0) = \tilde{T}_1(\hat{r}),$$

$$-\hat{K}_1 \frac{\partial \hat{T}_1}{\partial \hat{r}}(\mathcal{R}_1, \hat{x}) = 0,$$

$$\hat{T}_1(\mathcal{R}_0, \hat{x}) = \hat{T}_2(\mathcal{R}_0, \hat{x})$$

$$\mathcal{B} \frac{\partial \hat{T}_2}{\partial \hat{x}} + \left(\frac{\mathcal{C}(\hat{x})}{\hat{r}} - \frac{\hat{r}}{2} \frac{d\mathcal{B}}{d\hat{x}} \right) \frac{\partial \hat{T}_2}{\partial \hat{r}} = \frac{1}{P_2} \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left(\hat{r} \frac{\partial \hat{T}_2}{\partial \hat{r}} \right) \quad \mathcal{R}_0 \leq \hat{r} \leq \mathcal{R}_2,$$

$$\hat{T}_2(\hat{r}, 0) = \tilde{T}_2(\hat{r}),$$

$$-\hat{K}_2 \frac{\partial \hat{T}_2}{\partial \hat{r}}(\mathcal{R}_2, \hat{x}) = \bar{B} \hat{h}_2 \left(\hat{T}_2(\mathcal{R}_2, \hat{x}) - \hat{T}_{2,\infty} \right),$$

$$-\hat{K}_1 \frac{\partial \hat{T}_1}{\partial \hat{r}}(\mathcal{R}_0, \hat{x}) = -\hat{K}_2 \frac{\partial \hat{T}_2}{\partial \hat{r}}(\mathcal{R}_0, \hat{x})$$



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- Two-dimensional degree of crystallinity equation

$$B \frac{\partial Y_i}{\partial \hat{x}} + \left(\frac{C(\hat{x})}{\hat{r}} - \frac{\hat{r}}{2} \frac{dB}{d\hat{x}} \right) \frac{\partial Y_i}{\partial \hat{r}} = k_{Ai}(0) \exp(a_{2i} S_i^2) (Y_{\infty,i} - Y_i), \quad i = 1, 2,$$

$$Y_i(\hat{r}, 0) = \tilde{Y}_i(\hat{r}), \quad i = 1, 2,$$

- Effective dynamic viscosity

$$\hat{\mu}_{ei,0}(\hat{r}, \hat{x}) = \hat{G}_i \exp \left(\hat{E}_i (1 - \hat{T}_i) + \beta_i \left(\frac{Y_i}{Y_{\infty,i}} \right)^{n_i} \right) + \frac{2}{3} \alpha_i \hat{\lambda}_i S_i^2 \quad i = 1, 2.$$

- Cross-sectionally averaged effective dynamic viscosity

$$\langle \hat{\mu}_{e1,0} \rangle(\hat{x}) = \int_{\mathcal{R}_1}^{\mathcal{R}_0} \hat{\mu}_{e1,0}(\hat{r}, \hat{x}) \hat{r} d\hat{r} / \mathcal{A}_1,$$

$$\langle \hat{\mu}_{e2,0} \rangle(\hat{x}) = \int_{\mathcal{R}_0}^{\mathcal{R}_2} \hat{\mu}_{e2,0}(\hat{r}, \hat{x}) \hat{r} d\hat{r} / \mathcal{A}_2.$$



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- Two-dimensional equations for the molecular orientation tensor components

$$\mathcal{B} \frac{\partial S_{i rr}}{\partial \hat{x}} + \left(\frac{C(\hat{x})}{\hat{r}} - \frac{\hat{r}}{2} \frac{d\mathcal{B}}{d\hat{x}} \right) \frac{\partial S_{i rr}}{\partial \hat{r}} = (1 - S_{i rr})(1 + S_{i xx}) \frac{d\mathcal{B}}{d\hat{x}} - \frac{\phi_i}{\hat{\lambda}_i} \left\{ S_{i rr} + \frac{N_i}{3} [(S_{i rr} - 1)(S_{i rr} - O_{i s})] \right\}, \quad i = 1, 2,$$

$$\mathcal{B} \frac{\partial S_{i xx}}{\partial \hat{x}} + \left(\frac{C(\hat{x})}{\hat{r}} - \frac{\hat{r}}{2} \frac{d\mathcal{B}}{d\hat{x}} \right) \frac{\partial S_{i xx}}{\partial \hat{r}} = (1 + S_{i xx})(2 - S_{i xx}) \frac{d\mathcal{B}}{d\hat{x}} - \frac{\phi_i}{\hat{\lambda}_i} \left\{ S_{i xx} - \frac{N_i}{3} [(S_{i xx} + 1)(S_{i xx} - O_{i s})] \right\}, \quad i = 1, 2,$$

$$O_{i s}(\hat{r}, \hat{x}) = \frac{2}{3} (S_{i rr}^2 + S_{i xx}^2 - S_{i rr} S_{i xx}).$$

$$S_{i rr}(\hat{r}, 0) = -\tilde{S}_i(\hat{r}),$$

$$S_{i xx}(\hat{r}, 0) = 2\tilde{S}_i(\hat{r}).$$



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$(\hat{r}, \hat{x}) \mapsto (\xi, \eta)$ maps $\Omega_{\hat{r}\hat{x}} = \{[\mathcal{R}_1(\hat{x}), \mathcal{R}_2(\hat{x})] \times [0, 1]\}$ into a rectangular domain $\Omega_{\xi\eta} = \{[0, 1] \times [0, 1]\}$

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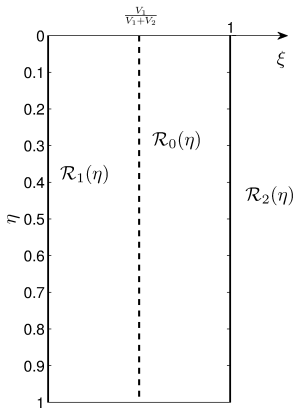
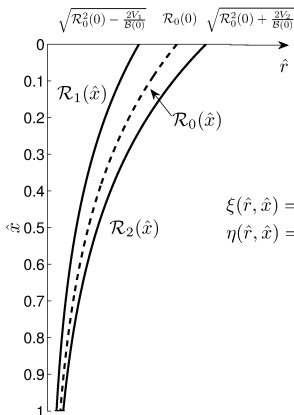
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$(\hat{r}, \hat{x}) \mapsto (\xi, \eta)$ maps $\Omega_{\hat{r}\hat{x}} = \{[\mathcal{R}_1(\hat{x}), \mathcal{R}_2(\hat{x})] \times [0, 1]\}$ into a rectangular domain $\Omega_{\xi\eta} = \{[0, 1] \times [0, 1]\}$



Two-dimensional equations of the 2D model

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- Two-dimensional energy equation

$$\frac{\partial \hat{T}_1}{\partial \eta} = \frac{1}{2(V_1 + V_2)} \frac{4}{\bar{P}_1} \frac{\partial}{\partial \xi} \left(\left(\xi + \frac{\mathcal{R}_1^2}{\mathcal{R}_2^2 - \mathcal{R}_1^2} \right) \frac{\partial \hat{T}_1}{\partial \xi} \right) \quad 0 \leq \xi \leq \frac{V_1}{V_1 + V_2},$$

$$\hat{T}_1(\xi, 0) = \tilde{T}_1(\xi),$$

$$-\hat{K}_1 \frac{\partial \hat{T}_1}{\partial \xi}(0, \eta) = 0,$$

$$\hat{T}_1\left(\frac{V_1}{V_1 + V_2}, \eta\right) = \hat{T}_2\left(\frac{V_1}{V_1 + V_2}, \eta\right)$$

$$\frac{\partial \hat{T}_2}{\partial \eta} = \frac{1}{2(V_1 + V_2)} \frac{4}{\bar{P}_2} \frac{\partial}{\partial \xi} \left(\left(\xi + \frac{\mathcal{R}_1^2}{\mathcal{R}_2^2 - \mathcal{R}_1^2} \right) \frac{\partial \hat{T}_2}{\partial \xi} \right) \quad \frac{V_1}{V_1 + V_2} \leq \xi \leq 1,$$

$$\hat{T}_2(\xi, 0) = \tilde{T}_2(\xi),$$

$$-\hat{K}_2 \frac{\partial \hat{T}_2}{\partial \xi}(1, \eta) = \frac{\mathcal{R}_2^2 - \mathcal{R}_1^2}{2\mathcal{R}_2} \bar{B} \hat{h}_2 \left(\hat{T}_2(1, \eta) - \hat{T}_2(\infty) \right),$$

$$-\hat{K}_1 \frac{\partial \hat{T}_1}{\partial \xi}\left(\frac{V_1}{V_1 + V_2}, \eta\right) = -\hat{K}_2 \frac{\partial \hat{T}_2}{\partial \xi}\left(\frac{V_1}{V_1 + V_2}, \eta\right)$$



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- Two-dimensional degree of crystallinity equation

$$\mathcal{B} \frac{\partial Y_i}{\partial \eta} = k_{Ai}(0) \exp(a_{2i} \mathcal{S}_i^2) (Y_{\infty,i} - Y_i), \quad i = 1, 2,$$

$$Y_i(\xi, 0) = \tilde{Y}_i(\xi), \quad i = 1, 2,$$

- Effective dynamic viscosity

$$\hat{\mu}_{ei,0}(\xi, \eta) = \hat{G}_i \exp\left(\hat{E}_i (1 - \hat{T}_i) + \beta_i \left(\frac{Y_i}{Y_{\infty,i}}\right)^{n_i}\right) + \frac{2}{3} \alpha_i \hat{\lambda}_i \mathcal{S}_i^2 \quad i = 1, 2.$$

- Cross-sectionally averaged effective dynamic viscosity

$$\langle \hat{\mu}_{e1,0} \rangle(\eta) = \frac{V_1 + V_2}{V_1} \int_0^{\frac{V_1}{V_1+V_2}} \hat{\mu}_{e1,0}(\xi, \eta) d\xi,$$

$$\langle \hat{\mu}_{e2,0} \rangle(\eta) = \frac{V_1 + V_2}{V_2} \int_{\frac{V_1}{V_1+V_2}}^1 \hat{\mu}_{e2,0}(\xi, \eta) d\xi.$$



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- Two-dimensional equations for the molecular orientation tensor components

$$B \frac{\partial S_{i rr}}{\partial \eta} = (1 - S_{i rr}) (1 + S_{i xx}) \frac{dB}{d\eta} - \frac{\phi_i}{\hat{\lambda}_i} \left\{ S_{i rr} + \frac{N_i}{3} [(S_{i rr} - 1)(S_{i rr} - O_{i s})] \right\}, \quad i = 1, 2,$$

$$B \frac{\partial S_{i xx}}{\partial \eta} = (1 + S_{i xx}) (2 - S_{i xx}) \frac{dB}{d\eta} - \frac{\phi_i}{\hat{\lambda}_i} \left\{ S_{i xx} - \frac{N_i}{3} [(S_{i xx} + 1)(S_{i xx} - O_{i s})] \right\}, \quad i = 1, 2,$$

$$O_{i s}(\xi, \eta) = \frac{2}{3} (S_{i rr}^2 + S_{i xx}^2 - S_{i rr} S_{i xx}).$$

$$S_{i rr}(\xi, 0) = -\tilde{S}_i(\xi),$$

$$S_{i xx}(\xi, 0) = 2\tilde{S}_i(\xi).$$



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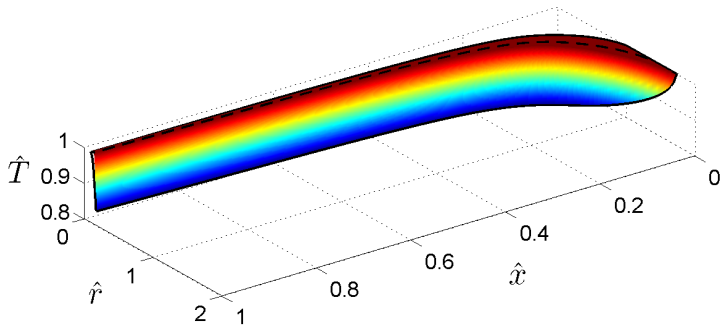
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$$E_2 = 10$$

Effect of the cladding's viscosity parameters

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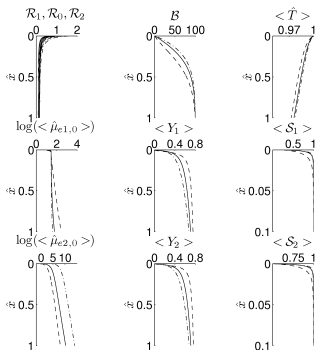
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$E_2 = 100$ (-), $E_2 = 50$ (- -) and $E_2 = 10$
(- · -)

$G_2 = 1$ (-), $G_2 = 0,01$ (- -) and $G_2 = 100$
(- · -)

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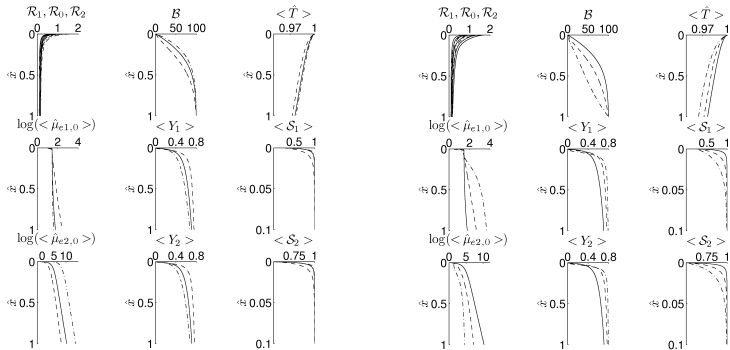
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1D model vs 2D model

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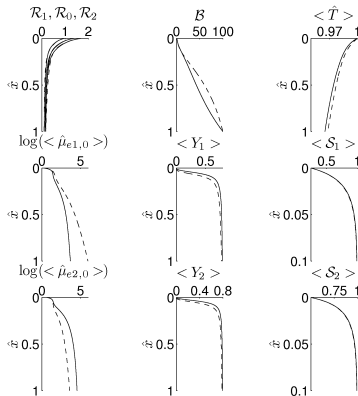
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1D (—) and 2D (---)



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Contributions of the present work:

- 1 Development of a quasi-two-dimensional model for semi-crystalline hollow compound fibers with modified Newtonian rheology.
- 2 1D model (asymptotic analysis for $\epsilon \ll 1$) for \mathcal{R}_i and \mathcal{B} and 2D equations for \hat{T}_i , \mathcal{S}_i and Y_i .
- 3 Determination of the two-dimensional fields of temperature, order parameter for molecular orientation and degree of crystallinity for hollow compound fibers.
- 4 Integro-differential model highly dependent on E_2 and G_2 .
- 5 Find substantial temperature non-uniformities (may affect the degree of crystallization and have great effects on the mechanical, optical,... properties of hollow compound fibers) in the radial direction.



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