Information Processing in Transmitting Recombination

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Abstract

The validity of a general template for transmitting recombination operators is established, and a sufficient condition to ensure the independence of the pieces of information manipulated in the process from the particulars of the operator is given.

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1 Introduction

Genetic algorithms (GAs) are heuristic search techniques based on the iterative generation of tentative solutions for a target problem [1]. These solutions are created by iteratively applying a set of operators to a pool of existing solutions (generated at random in the first place). Among these operators, recombination is given a central role in GAs. It consists of constructing a new solution by picking information from a pair of selected solutions (commonly termed parents), as well as possibly using some exogenous information. In this paper, we will focus on *transmitting* recombination, i.e., the construction of new solutions using only parental information. The validity of a general template of transmitting recombination will be established here, giving also a sufficient condition to ensure that information pieces manipulated during the process can be computed in advance. This will be done within the context of Forma Analysis [2].

2 Preliminaries

Let $\Phi = \{\phi_1, \cdots, \phi_n\}$ be a set of n independent equivalence relations defined over a discrete search space $\mathcal{S}$. Let Ξ_ϕ be the set of equivalence classes induced by ϕ, and let $[x]_\phi$ be the equivalence class to which x belong under ϕ. If it holds for Φ that for any two different solutions $x, y \in \mathcal{S}$, there exists $\phi \in \Phi$ such that $[x]_\phi \neq [y]_\phi$, then each solution $x \in \mathcal{S}$ can be represented as a string $\langle [x]_\phi \mid \phi \in \Phi \rangle$. Thus, $x = \langle \eta_1, \cdots, \eta_n \rangle \Longleftrightarrow \{x\} \triangleq \bigcap_{i=1}^n \eta_i$. Each of these equivalence classes η_i is a *basic forma* [2]. Equivalence relations are analogous to *genes*, and formae are analogous to *alleles*.

The *Dynastic Potential* $\Gamma(\{x,y\})$ of x and y is defined as $\Gamma(\{x,y\}) = \bigcap_{\phi\in\Phi}([x]_\phi \cup [y]_\phi)$. The *Similarity Set* $\Sigma(\{x,y\})$ is defined as $\Sigma(\{x,y\}) = \bigcap_{\phi\in\Phi, [x]_\phi=[y]_\phi} [x]_\phi$ (notice that $\Gamma(\{x,y\}) \subseteq \Sigma(\{x,y\})$). If for any formae η, ζ such that $\eta \cap \zeta \neq \emptyset$ (i.e., η and ζ are compatible), and any $x \in \eta$, $y \in \zeta$, it holds that $\eta \cap \zeta \cap \Sigma(\{x,y\}) \neq \emptyset$, then Φ is *separable*. If the intersection of any set of basic formae $\{\eta_1, \cdots, \eta_n\}$, $\eta_i \in \Xi_{\phi_i}$ is non-empty, then Φ is *orthogonal* (orthogonality implies separability, but the reverse is not true [2]). Let the notation $\xi \rhd \Psi$ denote that, given $\Psi = \cap_{j=1}^s \theta_j$, an index j exists such that $\xi \equiv \theta_j$, where ξ and θ_j are formae induced by the same equivalence relation $\phi \in \Phi$.

A recombination operator X can be defined as a function $X : \mathcal{S}\times\mathcal{S}\times\mathcal{S} \to [0,1]$, where $X(x,y,z)$ is the probability of generating z when x and y are recombined using X. A recombination operator X is said to be *transmitting* if, and only if, $\{z \mid X(x,y,z) > 0\} \subseteq \Gamma(\{x,y\})$.

Let $\eta \ni x$ be a basic forma. The dual forma $\varpi(\eta,x,y)$ is $\zeta \ni y$ if, and only if, $\phi \in \Phi$ exists such that $\eta, \zeta \in \Xi_\phi$, i.e., they are induced by the same equivalence relation ϕ.

3 Analyzing Transmitting Recombination

A transmitting recombination is process in which information is incrementally taken from the parents x and y to construct the descendant. It starts from a partially specified solution carrying features common to both parents, i.e., $\Psi_0 = \Sigma(\{x,y\})$. Subsequently, sets of gene values from any of the parents are selected and assigned to the descendant until a full solution is obtained. Each of these sets is called a *construction unit*. More formally, a construction unit $\Upsilon(\Psi,u,w)$ is an intersection of basic formae $\Theta \triangleq \cap_{j=1}^{g}\theta_j$, with $g \geq 1$, and $u \in \Theta$, such that $\Theta \cap \Psi \neq \emptyset$, and for any $\theta \triangleright \Theta$, it holds that $\theta \not\triangleright \Psi$, where Ψ is the partially specified solution, and u and w are the parents.

Construction units constitute the *information atoms* used to create the descendant, and their structure clearly depends on the particulars of the representation. In orthogonal representations $\Gamma(\{x,y\}) = \Gamma(\{\langle\eta_1,\cdots,\eta_n\rangle,\langle\zeta_1,\cdots,\zeta_n\rangle\}) \triangleq \prod_{i=1}^{n}\{\eta_i,\zeta_i\}$, i.e., the Cartesian product of all pairs $\{\eta_i,\zeta_i\}$, where $x \in \eta_i$, and $y \in \zeta_i$, for $1 \leq i \leq n$. Thus, it is possible to extend any partially specified solution using a single basic forma at a time, i.e., $\Upsilon(\Psi,u,w) = \sigma(\Psi,u)$, where $\sigma(\Psi,u) = [u]_{\phi_i}$ (with $i = \min\{j \mid \exists z,z' \in \Psi : z \in [u]_{\phi_j}, z' \notin [u]_{\phi_j}\}$) is the forma to which u belongs under the first unspecified gene (under any fixed arbitrary ordering) in Ψ. Thus, decisions reduce to either considering $\Upsilon(\Psi,u,w)$ or $\Upsilon(\Psi,w,u) = \varpi(\Upsilon(\Psi,u,w),u,w)$ in orthogonal representations. This is not the general case though: in many representations (e.g., the position-based representation of permutations [3]), choosing a certain forma at a given step may force the inclusion or exclusion of other formae in further steps. For this reason, construction units must be more complex. More precisely, we consider *compatibility sets* $K(\Psi,\eta,x,y)$ defined as the closure of the following expressions:

$$\eta \triangleright K(\Psi,\eta,x,y) \tag{1}$$

$$\left[\Gamma(\{x,y\}) \cap \Psi \cap K(\Psi,\eta,x,y) \cap \varpi(\eta',x,y) = \emptyset\right] \quad\Rightarrow\quad \eta' \triangleright K(\Psi,\eta,x,y), \tag{2}$$

i.e., the intersection of all formae η' ($x \in \eta'$) that must be included along with η to preserve feasibility within the dynastic potential. Thus, $\Upsilon(\Psi,u,w) = K(\Psi,\sigma(\Psi,u),u,w)$ in this context.

A naturally arising question is whether the process of constructing the descendant is in this situation analogous to the case of orthogonality, i.e., considering formae under an arbitrary ordering, and taking binary decisions between two compatibility sets at a time, $K(\Psi,\eta,x,y)$ and $K(\Psi,\zeta,y,x)$, where both $\eta \ni x$ and $\zeta \ni y$ are formae induced by ψ_i, and ψ_i is the first unspecified gene in Ψ. This is not a trivial question since compatibility sets are not symmetric as shown below:

Proposition 1. *$\eta \triangleright K(\Psi,\eta',u,w)$ does not imply that $\eta' \triangleright K(\Psi,\eta,u,w)$.*
Proof: By example. Let $\Phi = \{\phi_1,\cdots,\phi_n\}$, with $n \geq 2$. Let each equivalence relation ϕ_i induce two equivalence classes ϕ_i^0 and ϕ_i^1. Let $\phi_1^0 \cap \phi_2^1 = \emptyset$, and let $\phi_i^r \cap \phi_j^{r'} \neq \emptyset$, $i \neq j$, $r,r' \in \{0,1\}$ otherwise. Finally, let $x = \langle\phi_1^0,\phi_2^0,\cdots,\phi_n^k\rangle$ and $y = \langle\phi_1^1,\phi_2^1,\cdots,\phi_n^{k'}\rangle$.

The compatibility set of ϕ_1^0 is $K(\Psi,\phi_1^0,x,y) = \phi_1^0 \cap \phi_2^0$ since $\phi_1^0 \cap \varpi(\phi_2^0,x,y) = \phi_1^0 \cap \phi_2^1 = \emptyset$. However, the compatibility set of ϕ_2^0 is itself, since $\phi_2^0 \cap \phi_1^0 \neq \emptyset$ and $\phi_2^0 \cap \phi_1^1 \neq \emptyset$. Thus, $\phi_2^0 \triangleright K(\Psi,\phi_1^0,u,w)$ but $\phi_1^0 \not\triangleright K(\Psi,\phi_2^0,u,w)$. □

The construction units used to build the descendant are thus different depending upon the order in which the equivalence relations are considered, so this order might be relevant. However, any solution in the dynastic potential can be generated whatever this order be, as shown below:

Proposition 2. *Given $x, y \in \mathcal{S}$, any $z \in \Gamma(\{x, y\})$ can be generated by deciding between the compatibility sets of the alleles in x or y for any unspecified gene in the descendant.*
Proof: Let us assume that m decisions $\{\delta_1, \cdots, \delta_m\} \in \{0,1\}^m$ have been taken. Let decision δ_i mean that the descendant belongs to $\Delta(\delta_i, \Psi_{i-1}, x, y)$, where Ψ_j is the partially specified solution at step j, and $\Delta : \{0,1\} \times 2^{\mathcal{S}} \times \mathcal{S} \times \mathcal{S} \to 2^{\mathcal{S}}$ is defined as

$$\Delta(\delta, \Psi, u, w) = \begin{cases} K(\Psi, \sigma(\Psi, u), u, w) & \delta = 0 \\ K(\Psi, \sigma(\Psi, w), w, u) & \delta = 1 \end{cases} \tag{3}$$

Thus, $\Psi_0 = \Sigma(\{x, y\})$, and $\Psi_i \triangleq \Psi_{i-1} \cap \Delta(\delta_i, \Psi_{i-1}, x, y)$. Now, assume that there exists a solution $z = \langle \xi_1, \cdots, \xi_n \rangle \in \Gamma(\{x, y\})$ such that it cannot be generated by any sequence of binary decisions. We show that this is impossible because z can be made to belong to Ψ_i for $0 \leq i \leq m$.

The proof is done by induction on i. Initially, suppose $i = 0$. Since $\Psi_0 = \Sigma(\{x, y\})$, it is trivial that $z \in \Psi_0$. Next, the induction hypothesis is that for any $z \in \Gamma(\{x, y\})$, a sequence of decisions $\{\delta_1, \cdots, \delta_i\}$ exists such that z belongs to Ψ_i, $(i \leq k)$. Then, we consider the situation $i = k + 1$.

First of all, let ψ_j be the first unspecified equivalence relation in Ψ_k. Let $\xi_j \ni z$ be a forma induced by ψ_j, and let x be the parent belonging to ξ_j. Let us assume (to be proven absurd) that $z \notin \Psi_{k+1}$. Since, $z \in \Psi_k$ by the induction hypothesis, and $\Psi_{k+1} = \Psi_k \cap K(\Psi_k, \xi_j, x, y)$, it follows that $z \notin K(\Psi_k, \xi_j, x, y)$. This implies that some basic formae $\zeta_1, \cdots, \zeta_s$ exist such that $\zeta_r \triangleright K(\Psi_k, \xi_j, x, y)$, and $z \notin \zeta_r$, that is, $z \in \varpi(\zeta_r, x, y)$, for $1 \leq r \leq s$. Let $\Theta \ni z$ be the remaining formae in the compatibility set. According to the definition of compatibility set, it must be that

$$\bigcap_{r=1}^{s} \varpi(\zeta_r, x, y) \cap \Gamma(\{x, y\}) \cap \Psi_k \cap \Theta = \emptyset. \tag{4}$$

However, this intersection cannot be empty since z belong to every set involved in the above equation. We arrive at a contradiction and thus, $z \in \Psi_{k+1}$. Notice finally that Ψ_m must comprise a single solution (otherwise more decisions would be required). Hence, $\Psi_m = \{z\}$. $\square$

It thus suffices to consider equivalence relations in any arbitrary order, identifying in each step the first unspecified gene φ, and taking binary decisions on the compatibility sets of the formae to which the parents belong under φ. The next step is computing the compatibility sets involved in these decisions. A potential difficulty for computing them is the fact that they generally depend on the partially constructed solution so-far (the first parameter in $K(\cdot)$). However, compatibility sets are independent of this first parameter when the representation is separable, as shown below.

Proposition 3. $K(\Psi, \eta, x, y) = K(\Sigma(\{x, y\}), \eta, x, y)$ *in separable representations.*
Proof: The proof is done by induction on the number of compatibility sets considered in Ψ. Initially, $\Psi = \Sigma(\{x, y\})$, so the base case is trivial. Now, assume that $K(\Psi, \eta, x, y) = K(\Sigma(\{x, y\}), \eta, x, y)$ for separable representations whenever k compatibility sets are considered in Ψ. Subsequently, the $(k+1)$ case is examined. Assume that such a partially specified solution Ψ exists for which $K(\Psi, \eta, x, y) \neq K(\Sigma(\{x, y\}), \eta, x, y)$ where $x \in \eta$, $\eta \not\triangleright \Psi$, and $\varpi(\eta, x, y) \not\triangleright \Psi$. In this case, $K(\Psi, \eta, x, y) \subseteq K(\Sigma(\{x, y\}), \eta, x, y)$ since $\Psi \subseteq \Sigma(\{x, y\})$, i.e., there exists ξ such that $\xi \not\triangleright K(\Sigma(\{x, y\}), \eta, x, y)$, and $\xi \triangleright K(\Psi, \eta, x, y)$. Let $\Theta = K(\Sigma(\{x, y\}), \eta, x, y)$. Thus,

$$\text{(a)} \;\; \Gamma(\{x, y\}) \cap \Theta \cap \varpi(\xi, x, y) \neq \emptyset, \quad \text{and} \quad \text{(b)} \;\; \Gamma(\{x, y\}) \cap \Psi \cap \Theta \cap \varpi(\xi, x, y) = \emptyset. \tag{5}$$

Eq. (5a) implies that Θ and $\varpi(\xi, x, y)$ are compatible. It must also hold that $\Gamma(\{x, y\}) \cap \Psi \cap \varpi(\xi, x, y) \neq \emptyset$. If this were not true, ξ should be included in the compatibility set of a forma $\zeta \triangleright \Psi$ contradicting the induction hypothesis. Hence, Ψ and $\varpi(\xi, x, y)$ are compatible. For the same reason, it must be true that $\Gamma(\{x, y\}) \cap \Psi \cap \Theta \neq \emptyset$. Otherwise, and given that $K(\Psi, \eta, x, y) \subseteq \Theta$, it would mean that Ψ cannot be extended with η and hence $\varpi(\eta, x, y) \triangleright \Psi$. Thus, Ψ and Θ are compatible too. Now, consider two solutions v and w such that $v \in \Psi \cap \Theta$ and $w \in \Psi \cap \varpi(\xi, x, y)$. If the representation is separable, $\Sigma(\{v, w\}) \cap \Theta \cap \varpi(\xi, x, y) \neq \emptyset$. Let $\mathcal{I} = \Sigma(\{v, w\}) \cap \Theta \cap \varpi(\xi, x, y)$. Since $\Sigma(\{v, w\}) \subseteq \Psi$, Eq. (5b) implies that either $\mathcal{I} = \emptyset$ (i.e., the separability condition does not hold) or $\mathcal{I} \neq \emptyset$ and $\mathcal{I} \cap \Gamma(\{x, y\}) = \emptyset$. In the latter case, we have that for all $z \in \mathcal{I}$, exists at least one unspecified gene φ ($\{\zeta, \varpi(\zeta, x, y)\} \subseteq \Xi_\varphi$, $\zeta \not\triangleright \mathcal{I}$, $\varpi(\zeta, x, y) \not\triangleright \mathcal{I}$, $x \in \zeta$, $y \in \varpi(\zeta, x, y)$), such that z does not belong either to ζ or $\varpi(\zeta, x, y)$, i.e., $\mathcal{I} \cap \zeta = \mathcal{I} \cap \varpi(\zeta, x, y) = \emptyset$. However this implies that the separability condition does not hold, because $\varpi(\zeta, x, y)$ is compatible with Ψ (both x and y belong to Ψ), with Θ (otherwise, $\zeta \triangleright \Theta$, so φ would not be unspecified contradicting our hypothesis), and with $\varpi(\xi, x, y)$ (y belongs to $\varpi(\xi, x, y) \cap \varpi(\zeta, x, y)$). Since we arrive to a contradiction, there must not exist Ψ in separable representations for which $K(\Psi, \eta, x, y) \neq K(\Sigma(\{x, y\}), \eta, x, y)$. $\square$

Proposition 3 is important for it provides a sufficient condition to ensure that compatibility sets do not depend on decisions taken on-the-fly. Hence, they can be computed in advance at the beginning of the recombination process. The algorithm can subsequently handle them in the same way as single formae in orthogonal representations, i.e., as units that can be freely combined.

4 Conclusions

This work has studied the processing of information during transmitting recombination. Although we have focused on the GAs viewpoint, it must be noted that recombination also models standalone processes such as, for instance, fusing Bayesian networks into a consensus structure [4]. Thus, the concepts and principles presented in this paper have implications in a wider context than evolutionary computation.

Future work will be directed to a deeper study of non-separable representations. The structure of compatibility sets generally exhibits in this case a higher complexity. Additionally, trying to generalize the concepts presented in this paper to the so-called *multiparent recombination* (recombination with more that two parents) constitutes also a very interesting line for future developments.

References

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